Austrian Federal Competition For Advanced Students, Part 1, 2020

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by Ln142, parmenides51

1 Let x, y and z be positive real numbers such that $x \geq y+z$.
Proof that

$$\frac{x+y}{z}+\frac{y+z}{x}+\frac{z+x}{y} \geq 7$$

When does equality occur?

(Walther Janous)

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2 Let ABC be a right triangle with a right angle in C and a circumcenter U. On the sides AC and BC, the points D and E lie in such a way that $\angle EUD = 90^o$. Let F and G be the projection of D and E on AB, respectively. Prove that FG is half as long as AB.

(Walther Janous)

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3 On a blackboard there are three positive integers. In each step the three numbers on the board are denoted as a, b, c such that $a > gcd(b, c)$, then a gets replaced by $a - gcd(b, c)$. The game ends if there is no way to denote the numbers such that $a > gcd(b, c)$.

Prove that the game always ends and that the last three numbers on the blackboard only depend on the starting numbers.

(Theresia Eisenkölbl)

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4 Determine all positive integers N such that

$$2^N - 2N$$

is a perfect square.

(Walther Janous)
