Cono Sur Olympiad 2020

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–	Day 1
1	Ari and Beri play a game using a deck of 2020 cards with exactly one card with each number from 1 to 2020. Ari gets a card with a number a and removes it from the deck. Beri sees the card, chooses another card from the deck with a number b and removes it from the deck. Then Beri writes on the board exactly one of the trinomials x^2-ax+b or x^2-bx+a from his choice. This process continues until no cards are left on the deck. If at the end of the game every trinomial written on the board has integer solutions, Beri wins. Otherwise, Ari wins. Prove that Beri can always win, no matter how Ari plays.
2	Given 2021 distinct positive integers non divisible by 2^{1010}, show that it's always possible to choose 3 of them a, b and c, such that $\|b^2-4ac\|$ is not a perfect square.
3	Let ABC be an acute triangle such that $AC < BC$ and ω its circumcircle. M is the midpoint of BC. Points F and E are chosen in AB and BC, respectively, such that $AC = CF$ and $EB = EF$. The line AM intersects ω in $D \neq A$. The line DE intersects the line FM in G. Prove that G lies on ω.
–	Day 2
4	Let ABC be an acute scalene triangle. D and E are variable points in the half-lines AB and AC (with origin at A) such that the symmetric of A over DE lies on BC. Let P be the intersection of the circles with diameter AD and AE. Find the locus of P when varying the line segment DE.
5	There is a pile with 15 coins on a table. At each step, Pedro choses one of the piles in the table with $a>1$ coins and divides it in two piles with $b\geq 1$ and $c \geq 1$ coins and writes in the board the product abc. He continues until there are 15 piles with 1 coin each. Determine all possible values that the final sum of the numbers in the board can have.
6	A 4 x 4 square board is called $brasuca$ if it follows all the conditions: each box contains one of the numbers $0, 1, 2, 3, 4$ or 5; the sum of the numbers in each line is 5; the sum of the numbers in each column is 5; the sum of the numbers on each diagonal of four squares is 5; the number written in the upper left box of the board is less than or equal to the other numbers the board;

when dividing the board into four 2×2 squares, in each of them the sum of the four numbers is 5.

How many $"brasucas"$ boards are there?