2021 Hong Kong (China) Mathematical Olympiad

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1 There is a table with n rows and 18 columns. Each of its cells contains a 0 or a 1. The table satisfies the following properties:

-Every two rows are different.
-Each row contains exactly 6 cells that contain 1.
-For every three rows, there exists a column so that the intersection of the column with the three rows (the three cells) all contain 0.

What is the greatest possible value of n?

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2 For each positive integer n larger than 1 with prime factorization $p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$, its *signature* is defined as the sum $\alpha_1+\alpha_2+\cdots+\alpha_k$. Does there exist 2020 consecutive positive integers such that among them, there are exactly 1812 integers whose signatures are strictly smaller than 11?

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3 Let $ABCD$ be a cyclic quadrilateral inscribed in a circle Γ such that $AB = AD$. Let E be a point on the segment CD such that $BC = DE$. The line AE intersect Γ again at F. The chords AC and BF meet at M. Let P be the symmetric point of C about M. Prove that PE and BF are parallel.

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4 Let a, b and c be positive real numbers satisfying $abc = 1$. Prove that

$$\frac{1}{a^3+2b^2+2b+4}+\frac{1}{b^3+2c^2+2c+4}+\frac{1}{c^3+2a^2+2a+4}\le\frac{1}{3}.$$
