Austrian Mathematical Olympiad Junior Regional Competition 2019

www.artofproblemsolving.com/community/c1666589

by parmenides51

1 Let x and y be integers with $x+y \neq 0$. Find all pairs (x, y) such that

$$\frac{x^2+y^2}{x+y}=10.$$

(Walther Janous)

---

2 A square $ABCD$ is given. Over the side BC draw an equilateral triangle BCD on the outside. The midpoint of the segment AS is N and the midpoint of the side CD is H. Prove that $\angle NHC = 60^o$.

.

(Karl Czakler)

---

3 Alice and Bob are playing a year number game.

There will be two game numbers 19 and 20 and one starting number from the set $\{9, 10\}$ used. Alice chooses independently her game number and Bob chooses the starting number. The other number is given to Bob. Then Alice adds her game number to the starting number, Bob adds his game number to the result, Alice adds her number of games to the result, etc. The game continues until the number 2019 is reached or exceeded.

Whoever reaches the number 2019 wins. If 2019 is exceeded, the game ends in a draw. • Show that Bob cannot win. • What starting number does Bob have to choose to prevent Alice from winning?

(Richard Henner)

---

4 Let p, q, r and s be four prime numbers such that

$$5 < p < q < r < s < p + 10.$$

Prove that the sum of the four prime numbers is divisible by 60.

(Walther Janous)
