

**Uzbekistan National Olympiad 2016**

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- 1  $\omega$  is circumcircle of triangle  $ABC$  and  $BB_1, CC_1$  are bisectors of  $ABC$ .  $I$  is center incircle.  $B_1C_1$  and  $\omega$  intersects at  $M$  and  $N$ . Find the ratio of circumradius of  $ABC$  to circumradius  $MIN$ .

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- 2  $n$  is natural number and  $p$  is prime number. If  $1 + np$  is square of natural number then prove that  $n + 1$  is equal to some sum of  $p$  square of natural numbers.

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- 3 In triangle  $ABC$   $\omega$  is incircle and  $\omega_1, \omega_2, \omega_3$  is tangents to  $\omega$  and two sides of  $ABC$ .  $r, r_1, r_2, r_3$  is radius of  $\omega, \omega_1, \omega_2, \omega_3$ . Prove that  $\sqrt{r_1 r_2} + \sqrt{r_2 r_3} + \sqrt{r_3 r_1} = r$

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- 4  $a, b, c, x, y, z$  are positive real numbers and  $bz + cy = a, az + cx = b, ay + bx = c$ . Find the least value of following function  $f(x, y, z) = \frac{x^2}{1+x} + \frac{y^2}{1+y} + \frac{z^2}{1+z}$

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- 5 Solve following system equations:

$$\begin{cases} 3x + 4y = 26 \\ \sqrt{x^2 + y^2 - 4x + 2y + 5} + \sqrt{x^2 + y^2 - 20x - 10y + 125} = 10 \end{cases}$$