AoPS Online
Art of Problem Solving

Argentine National Olympiad 2020

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by parmenides51

–	Level 3	
–	Day 1	
1	For every positive integer n, let $S(n)$ be the sum of the digits of n. Find, if any, a 171-digit positive integer n such that 7 divides $S(n)$ and 7 divides $S(n+1)$.	
2	Let $k \geq 1$ be an integer. Determine the smallest positive integer n such that some cells on an $n \times n$ board can be painted black so that in each row and in each column there are exactly k black cells, and furthermore, the black cells do not share a side or a vertex with another black square. Clarification: You have to answer n based on k.	
3	Let ABC be a right isosceles triangle with right angle at A. Let E and F be points on AB and AC respectively such that $\angle ECB = 30^o$ and $\angle FBC = 15^o$. Lines CE and BF intersect at P and line AP intersects side BC at D. Calculate the measure of angle $\angle FDC$.	
–	Day 2	
4	Let a and b be positive integers such that $\frac{5a^4+a^2}{b^4+3b^2+4}$ is an integer. Show that a is not prime.	
5	Determine the highest possible value of: $$S = a_1a_2a_3 + a_4a_5a_6 + ... + a_{2017}a_{2018}a_{2019} + a_{2020}$$ where $(a_1, a_2, a_3, ..., a_{2020})$ is a permutation of $(1, 2, 3, ..., 2020)$. Clarification: In S, each term, except the last one, is the multiplication of three numbers.	
6	Let $n \geq 3$ be an integer. Lucas and Matías play a game in a regular n-sided polygon with a vertex marked as a trap. Initially Matías places a token at one vertex of the polygon. In each step, Lucas says a positive integer and Matías moves the token that number of vertices clockwise or counterclockwise, at his choice. a) Determine all the $n \geq 3$ such that Matías can locate the token and move it in such a way as to never fall into the trap, regardless of the numbers Lucas says. Give the strategy to Matías. b) Determine all the $n \geq 3$ such that Lucas can force Matías to fall into the trap. Give the strategy to Lucas.	

Note. The two players know the value of n and see the polygon.