2016 Istek Lyceum Math Olympiad

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1 Find all functions $f : \mathbb{R} \to \mathbb{R}$ for which

$$f(x+y) = f(x-y) + f(f(1-xy))$$

holds for all real numbers x and y

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2 Let ω be the semicircle with diameter PQ. A circle k is tangent internally to ω and to segment PQ at C. Let AB be the tangent to K perpendicular to PQ, with A on ω and B on the segment CQ. Show that AC bisects angle $\angle PAB$

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3 Let n, m and k be positive integers satisfying $(n-1)n(n+1) = m^k$. Prove that $k = 1$.

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4 Zeroes are written in all cells of a 5×5 board. We can take an arbitrary cell and increase by 1 the number in this cell and the cells having a common side with it. Is it possible to obtain the number 2012 in all cells simultaneously?
