

South Africa National Olympiad 2020

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by DylanN

- 1 Find the smallest positive multiple of 20 with exactly 20 positive divisors.
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- 2 Let S be a square with sides of length 2 and R be a rhombus with sides of length 2 and angles measuring 60° and 120° . These quadrilaterals are arranged to have the same centre and the diagonals of the rhombus are parallel to the sides of the square. Calculate the area of the region on which the figures overlap.
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- 3 If x, y, z are real numbers satisfying

$$\begin{aligned}(x+1)(y+1)(z+1) &= 3 \\(x+2)(y+2)(z+2) &= -2 \\(x+3)(y+3)(z+3) &= -1,\end{aligned}$$

find the value of

$$(x+20)(y+20)(z+20).$$

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- 4 A positive integer k is said to be *visionary* if there are integers $a > 0$ and $b \geq 0$ such that $a \cdot k + b \cdot (k+1) = 2020$. How many visionary integers are there?
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- 5 Let ABC be a triangle, and let T be a point on the extension of AB beyond B , and U a point on the extension of AC beyond C , such that $BT = CU$. Moreover, let R and S be points on the extensions of AB and AC beyond A such that $AS = AT$ and $AR = AU$. Prove that R, S, T, U lie on a circle whose centre lies on the circumcircle of ABC .
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- 6 Marjorie is the drum major of the world's largest marching band, with more than one million members. She would like the band members to stand in a square formation. To this end, she determines the smallest integer n such that the band would fit in an $n \times n$ square, and lets the members form rows of n people. However, she is dissatisfied with the result, since some empty positions remain. Therefore, she tells the entire first row of n members to go home and repeats the process with the remaining members. Her aim is to continue it until the band forms a perfect square, but as it happens, she does not succeed until the last members are sent home. Determine the smallest possible number of members in this marching band.
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