beroAmerican 2015

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–	Day 1	
1	The number 125 can be written as a sum of some pairwise coprime integers larger than 1. Determine the largest number of terms that the sum may have.
2	A line r contains the points A, B, C, D in that order. Let P be a point not in r such that $\angle APB = \angle CPD$. Prove that the angle bisector of $\angle APD$ intersects the line r at a point G such that: $\frac{1}{GA}+\frac{1}{GC}=\frac{1}{GB}+\frac{1}{GD}$
3	Let α and β be the roots of x^2-qx+1, where q is a rational number larger than 2. Let $s_1=\alpha+\beta$, $t_1=1$, and for all integers $n\geq 2$: $s_n=\alpha^n+\beta^n$ $t_n=s_{n-1}+2s_{n-2}+\cdots+(n-1)s_1+n$ Prove that, for all odd integers n, t_n is the square of a rational number.
–	Day 2
4	Let ABC be an acute triangle and let D be the foot of the perpendicular from A to side BC. Let P be a point on segment AD. Lines BP and CP intersect sides AC and AB at E and F, respectively. Let J and K be the feet of the peroendiculars from E and F to AD, respectively. Show that $\frac{FK}{KD}=\frac{EJ}{JD}$.
5	Find all pairs of integers (a,b) such that $(b^2+7(a-b))^2=a^3b$.
6	Beto plays the following game with his computer: initially the computer randomly picks 30 integers from 1 to 2015, and Beto writes them on a chalkboard (there may be repeated numbers). On each turn, Beto chooses a positive integer k and some if the numbers written on the chalkboard, and subtracts k from each of the chosen numbers, with the condition that the resulting numbers remain non-negative. The objective of the game is to reduce all 30 numbers to 0, in which case the game ends. Find the minimal number n such that, regardless of which numbers the computer chooses, Beto can end the game in at most n turns.