Mathematical Olympiad 2010

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by cjquines0

1 Find all primes that can be written both as a sum of two primes and as a difference of two primes.

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2 On a cyclic quadrilateral $ABCD$, there is a point P on side AD such that the triangle CDP and the quadrilateral $ABCP$ have equal perimeters and equal areas. Prove that two sides of $ABCD$ have equal lengths.

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3 Let $\mathbb{R}^*$ be the set of all real numbers, except 1. Find all functions $f : \mathbb{R}^* \to \mathbb{R}$ that satisfy the functional equation

$$x + f(x) + 2f\left(\frac{x+2009}{x-1}\right) = 2010$$

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4 There are 2008 blue, 2009 red and 2010 yellow chips on a table. At each step, one chooses two chips of different colors, and recolor both of them using the third color. Can all the chips be of the same color after some steps? Prove your answer.

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5 Determine, with proof, the smallest positive integer n with the following property: For every choice of n integers, there exist at least two whose sum or difference is divisible by 2009.
