2015 TST for EGMO in Serbia

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– Find all polynomials $P(x)$ such that for every real x it hold $(x+100)P(x)-xP(x+1)=1$.

– Let $ABCD$ be cyclic quadriateral and let AC and BD intersect at E and AB and CD at F. Let K be point in plane such that $ABKC$ is parallelogram. Prove $\angle AFE = \angle CDF$.

– Define $corner$ as a 'broken' line(in Cartesian coordinate plane) consisting of one vertical and one horizontal line, with $ends$ at first point and last point of 'broken' line (for example ABC is corner if B is in plane such that $AB \perp BC$ and $AB||x$ or $AB||y$ (note that in following statement one chooses one of such B)). In Cartesian coordinate plane there are n blue and n red points with all different x and y coordinates. Prove that one can draw n $corners$ without common points such that every $corner$ has one blue and one red end.

– Let $a_n{}_1^\infty$ be array such that $a_1 = 2$ and for every $n \geq 1$ $a_{n+1} = 2^{a_n} + 2$. Let m, n be positive integers such that $m < n$. Prove that $a_m | a_n$.
