Turkey Junior National Olympiad 2015

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1   For a non-constant function $f : \mathbb{R} \to \mathbb{R}$ prove that there exist real numbers x, y satisfying $f(x + y) < f(xy)$

2   In an exhibition there are 100 paintings each of which is made with exactly k colors. Find the minimum possible value of k if any 20 paintings have a common color but there is no color that is used in all paintings.

3   Find all pairs (p, n) so that p is a prime number, n is a positive integer and

$$p^3 - 2p^2 + p + 1 = 3^n$$

holds.

4   Let ABC be a triangle and D be the midpoint of the segment BC. The circle that passes through D and tangent to AB at B, and the circle that passes through D and tangent to AC at C intersect at $M \neq D$. Let M' be the reflection of M with respect to BC. Prove that M' is on AD.