Turkey Junior National Olympiad 2020

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1 Determine all real number (x, y) pairs that satisfy the equation.

$$2x^2 + y^2 + 7 = 2(x + 1)(y + 1)$$

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2 If the ratio

$$\frac{17m + 43n}{m - n}$$

is an integer where m and n positive integers, let's call (m, n) is a special pair. How many numbers can be selected from $1, 2, ..., 2021$, any two of which do not form a special pair?

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3 The circumcenter of an acute-triangle ABC with $|AB| < |BC|$ is O, D and E are midpoints of $|AB|$ and $|AC|$, respectively. OE intersects BC at K, the circumcircle of OKB intersects OD second time at L. F is the foot of altitude from A to line KL. Show that the point F lies on the line DE

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4 There are dwarves in a forest and each one of them owns exactly 3 hats which are numbered with numbers $1, 2, \ldots 28$. Three hats of a dwarf are numbered with different numbers and there are 3 festivals in this forest in a day. In the first festival, each dwarf wears the hat which has the smallest value, in the second festival, each dwarf wears the hat which has the second smallest value and in the final festival each dwarf wears the hat which has the biggest value. After that, it is realized that there is no dwarf pair such that both of two dwarves wear the same value in at least two festivals. Find the maximum value of number of dwarves.
