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– level 2

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**1** We say that a positive integer is super odd if all of its digits are odd. For example, 1737 is super odd and 3051 is not. Find an even positive integer that cannot be express as a sum of two super odd numbers and explain why it is not possible to express it thus.

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**2** a) Determine if there are positive integers  $a, b$  and  $c$ , not necessarily distinct, such that  $a + b + c = 2020$  and  $2^a + 2^b + 2^c$  it's a perfect square.  
b) Determine if there are positive integers  $a, b$  and  $c$ , not necessarily distinct, such that  $a + b + c = 2020$  and  $3^a + 3^b + 3^c$  it's a perfect square.

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**3** There is a box with 2020 stones. Ana and Beto alternately play removing stones from the box and starting with Ana. Each player in turn must remove a positive number of stones that is capicua. Whoever leaves the box empty wins. Determine which of the two has a strategy winner and explain what that strategy is. *Note* : A positive integer is capicua if it can be read equally from right to right. left and left to right. For example, 3, 22, 484 and 2002 are capicuas.

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**4** Let  $ABC$  be a right triangle, right at  $B$ , and let  $M$  be the midpoint of the side  $BC$ . Let  $P$  be the point in bisector of the angle  $\angle BAC$  such that  $PM$  is perpendicular to  $BC$  ( $P$  is outside the triangle  $ABC$ ). Determine the triangle area  $ABC$  if  $PM = 1$  and  $MC = 5$ .

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**5** We say that a positive integer  $n$  is circular if it is possible to place the numbers  $1, 2, \dots, n$  in a circumference so that there are no three adjacent numbers whose sum is a multiple of 3.  
a) Show that 9 is not circular  
b) Show that any integer greater than 9 is circular.

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– level 1

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**1** Sofia places the dice on a table as shown in the figure, matching faces that have the same number on each die. She circles the table without touching the dice. What is the sum of the numbers of all the faces that she cannot see? *Note*. In all given the numbers on the opposite faces add up to 7.

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**2** Paul wrote the list of all four-digit numbers such that the hundreds digit is 5 and the tens digit is 7. For example, 1573 and 7570 are on Paul's list, but 2754 and 571 are not. Find the sum of all the numbers on Pablo's list. *Note*. The numbers on Pablo's list cannot start with zero.

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- 3** A clueless ant makes the following route: starting at point  $A$  goes 1 cm north, then 2 cm east, then 3 cm south, then 4 cm west, immediately 5 cm north, continues 6 cm east, and so on, finally 41 cm north and ends in point  $B$ . Calculate the distance between  $A$  and  $B$  (in a straight line).
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- 4** Maria has a  $6 \times 5$  board with some shaded squares, as in the figure. She writes, in some order, the digits 1, 2, 3, 4 and 5 in the first row and then completes the board as follows: look at the number written in the shaded box and write the number that occupies the position indicated by the box shaded as the last number in the next row, and repeat the other numbers in the first four squares, following the same order as in the previous row.  
For example, if you wrote 2, 3, 4, 1, 5 in the first row, then since 4 is in the shaded box, the number that occupies the fourth place (1) is written in the last box of the second row and completes it with the remaining numbers in the order in which they were. She remains: 2, 3, 4, 5, 1.  
Then, to complete the third row, as in the shaded box is 3, the number located in the third place (4) writes it in the last box and gets 2, 3, 5, 1, 4. Following in the same way, he gets the board of the figure.  
Show a way to locate the numbers in the first row to get the numbers 2, 4, 5, 1, 3 in the last row.
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- 5** On a table there are several cards, some face up and others face down. The allowed operation is to choose 4 cards and turn them over. The goal is to get all the cards in the same state (all face up or all face down). Determine if the objective can be achieved through a sequence of permitted operations if initially there are:  
a) 101 cards face up and 102 face down;  
b) 101 cards face up and 101 face down.
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