Art of Problem Solving
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- $\quad$ level 2

1 A positive integer is called piola if the 9 is the remainder obtained by dividing it by $2,3,4,5,6,7,8,9$ and 10 and it's digits are all different and nonzero. How many piolas are there between 1 and 100000?

2 There is a board with 2020 squares in the bottom row and 2019 in the top row, located as shown shown in the figure.
https://cdn.artofproblemsolving.com/attachments/f/3/516ad5485c399427638c3d1783593d79d830c
png
In the bottom row the integers numbers from 1 to 2020 are placed in some order. Then in each box in the top row records the multiplication of the two numbers below it. How can they place the numbers in the bottom row so that the sum of the numbers in the top row be the smallest possible?

3 On the sides $A B, B C$ and $C A$ of a triangle $A B C$ are located the points $P, Q$ and $R$ respectively, such that $B Q=2 Q C, C R=2 R A$ and $\angle P R Q=90^{\circ}$. Show that $\angle A P R=\angle R P Q$.

4 Find the smallest positive integer $N$ of two or more digits that has the following property: If we insert any non-null digit $d$ between any two adjacent digits of $N$ we obtain a number that is a multiple of $d$.

5 We consider the $n$ vertices of a regular polygon with $n$ sides. There is a set of triangles with vertices at these $n$ points with the property that for each triangle in the set, the sides of at least one are not the side of any other triangle in the set. What is the largest amount of triangles that can have the set?

Consideramos los $n$ vértices de un polígono regular de $n$ lados. Se tiene un conjunto de triángulos con vértices en estos $n$ puntos con la propiedad que para cada triángulo del conjunto, al menos uno
de sus lados no es lado de ningún otro triángulo del conjunto. ¿Cuál es la mayor cantidad de triángulos que puede tener el conjunto?

- $\quad$ level 1

1 Find all the two-digit numbers $\overline{a b}$ that squared give a result where the last two digits are $\overline{a b}$.

2 More than five competitors participated in a chess tournament. Each competitor played exactly once against each of the other competitors. Five of the competitors they each lost exactly two games. All other competitors each won exactly three games. There were no draws in the tournament. Determine how many competitors there were and show a tournament that verifies all conditions.

3 Gus has to make a list of 250 positive integers, not necessarily distinct, such that each number is equal to the number of numbers in the list that are different from it. For example, if 15 is a number from the list so the list contains 15 numbers other than 15 . Determine the maximum number of distinct numbers the Gus list can contain.

4 You have to divide a square paper into three parts, by two straight cuts, so that by locating these parts properly, without gaps or overlaps, an obtuse triangle is formed. Indicate how to cut the square and how to assemble the triangle with the three parts.

5 There is a board with three rows and 2019 columns. In the first row are written the numbers integers from 1 to 2019 inclusive, ordered from smallest to largest. In the second row, Ana writes those same numbers but ordered at your choice. In each box in the third row write the difference between the two numbers already written in the same column (the largest minus the smallest). Beto have to paint some numbers in the third row so that the sum of the numbers painted is equal to the sum of the numbers in that row that were left unpainted. Can Ana complete the second row so that Beto does not achieve his goal?

