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1 Each of the numbers $1, 2, \dots, 10$ is colored red or blue. 5 is red and at least one number is blue. If m, n are different colors and $m + n \leq 10$, then $m + n$ is blue. If m, n are different colors and $mn \leq 10$, then mn is red. Find all the colors.

2 $p(x)$ is a polynomial such that $p(y^2 + 1) = 6y^4 - y^2 + 5$. Find $p(y^2 - 1)$.

3 Are there any integral solutions to $n^2 + (n + 1)^2 + (n + 2)^2 = m^2$?

4 The vertices of a triangle are three-dimensional lattice points. Show that its area is at least $\frac{1}{2}$.

5 Let $f(n)$ be defined on the positive integers and satisfy: $f(\text{prime}) = 1$, $f(ab) = af(b) + f(a)b$. Show that f is unique and find all n such that $n = f(n)$.

6 Solve

$$\begin{cases} y(x + y)^2 = 9 \\ y(x^3 - y^3) = 7 \end{cases}$$
