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by parmenides51

- 1 Find all polynomials  $f(x)$  such that  $f(2x) = f'(x)f''(x)$ .

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- 2  $ABCD$  is a square side 1.  $P$  and  $Q$  lie on the side  $AB$  and  $R$  lies on the side  $CD$ . What are the possible values for the circumradius of  $PQR$ ?

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- 3 Find all pairs  $(m, n)$  of integers such that  $n^2 - 3mn + m - n = 0$ .

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- 4 Which of the following statements are true?  
(A)  $X$  implies  $Y$ , or  $Y$  implies  $X$ , where  $X$  is the statement, the lines  $L_1, L_2, L_3$  lie in a plane, and  $Y$  is the statement, each pair of the lines  $L_1, L_2, L_3$  intersect.  
(B) Every sufficiently large integer  $n$  satisfies  $n = a^4 + b^4$  for some integers  $a, b$ .  
(C) There are real numbers  $a_1, a_2, \dots, a_n$  such that  $a_1 \cos x + a_2 \cos 2x + \dots + a_n \cos nx > 0$  for all real  $x$ .

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- 5 Find the largest cube which can be placed inside a regular tetrahedron with side 1 so that one of its faces lies on the base of the tetrahedron.

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