www.artofproblemsolving.com/community/c1971550
by parmenides51

1 Find the side lengths of the triangle ABC with area S and $\angle BAC = x$ such that the side BC is as short as possible.

---

2 Find all positive integers m, n such that $n + (n + 1) + (n + 2) + ... + (n + m) = 1000$.

---

3 Find a polynomial with integer coefficients which has $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}+\sqrt[3]{3}$ as roots.

---

4 Points $H_1, H_2, ..., H_n$ are arranged in the plane so that each distance $H_iH_j \le 1$. The point P is chosen to minimise $\max(PH_i)$. Find the largest possible value of $\max(PH_i)$ for $n = 3$. Find the best upper bound you can for $n = 4$.

---

5 $a_1, a_2, ..., a_n$ are constants such that $f(x) = 1 + a_1 cos x + a_2 cos 2x + ... + a_n cos nx \ge 0$ for all x. We seek estimates of a_1. If $n = 2$, find the smallest and largest possible values of a_1. Find corresponding estimates for other values of n.
