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by parmenides51

1 The feet of the altitudes in the triangle ABC are A', B', C'. Find the angles of $A'B'C'$ in terms of the angles A, B, C. Show that the largest angle in $A'B'C'$ is at least as big as the largest angle in ABC. When is it equal?

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2 Find all positive integers m, n such that $m^3 - n^3 = 999$.

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3 Show that for every real $x \ge \frac{1}{2}$ there is an integer n such that $|x - n^2| \le \sqrt{x - \frac{1}{4}}$.

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4 Find constants $A > B$ such that $\frac{f\left(\frac{1}{1+2x}\right)}{f(x)}$ is independent of x, where $f(x) = \frac{1+Ax}{1+Bx}$ for all real $x \ne -\frac{1}{B}$. Put $a_0 = 1$, $a_{n+1} = \frac{1}{1+2a_n}$. Find an expression for an by considering $f(a_0), f(a_1), ...$

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5 Let S be the set of all real polynomials $f(x) = ax^3 + bx^2 + cx + d$ such that $|f(x)| \le 1$ for all $-1 \le x \le 1$. Show that the set of possible $|a|$ for f in S is bounded above and find the smallest upper bound.
