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by parmenides51

1 Let $\{x\}$ denote the fractional part of x, $x - [x]$. The sequences $x_1, x_2, x_3, ...$ and $y_1, y_2, y_3, ...$ are such that $\lim\{x_n\} = \lim\{y_n\} = 0$. Is it true that $\lim\{x_n + y_n\} = 0$? $\lim\{x_n - y_n\} = 0$?

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2 $a_1 + a_2 + ... + a_n = 0$, for some k we have $a_j \le 0$ for $j \le k$ and $a_j \ge 0$ for $j > k$. If ai are not all 0, show that $a_1 + 2a_2 + 3a_3 + ... + na_n > 0$.

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3 Show that an integer $= 7 \mod 8$ cannot be sum of three squares.

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4 Let $f(x) = 1 + \frac{2}{x}$. Put $f_1(x) = f(x)$, $f_2(x) = f(f_1(x))$, $f_3(x) = f(f_2(x))$, ... Find the solutions to $x = f_n(x)$ for $n > 0$.

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5 Let $f(r)$ be the number of lattice points inside the circle radius r, center the origin.
Show that $\lim_{r\to\infty} \frac{f(r)}{r^2}$ exists and find it. If the limit is k, put $g(r) = f(r) - kr^2$.
Is it true that $\lim_{r\to\infty} \frac{g(r)}{r^h} = 0$ for any $h < 2$?
