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by parmenides51

1 p parallel lines are drawn in the plane and q lines perpendicular to them are also drawn. How many rectangles are bounded by the lines?

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2 You are given a ruler with two parallel straight edges a distance d apart. It may be used
(1) to draw the line through two points,
(2) given two points a distance $\geq d$ apart, to draw two parallel lines, one through each point,
(3) to draw a line parallel to a given line, a distance d away.
One can also (4) choose an arbitrary point in the plane,
and (5) choose an arbitrary point on a line.
Show how to construct :
(A) the bisector of a given angle, and
(B) the perpendicular to the midpoint of a given line segment.

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3 Show that there are only finitely many triples (a, b, c) of positive integers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{1000}$.

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4 The sequence $a_1, a_2, a_3, ...$ of positive reals is such that $\sum a_i$ diverges.
Show that there is a sequence $b_1, b_2, b_3, ...$ of positive reals such that $\lim b_n = 0$ and $\sum a_ib_i$ diverges.

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5 $a_1, a_2, a_3, ...$ are positive reals such that $a_n^2 \geq a_1 + a_2 + ... + a_{n-1}$.
Show that for some $C > 0$ we have $a_n \geq Cn$ for all n.

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6 The vertices of a triangle are lattice points. There are no lattice points on the sides (apart from the vertices) and n lattice points inside the triangle. Show that its area is $n+\frac{1}{2}$. Find the formula for the general case where there are also m lattice points on the sides (apart from the vertices).
