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- 1 Find the maximum and minimum values of $x^2 + 2y^2 + 3z^2$ for real x, y, z satisfying $x^2 + y^2 + z^2 = 1$.

- 2 How many different ways (up to rotation) are there of labeling the faces of a cube with the numbers $1, 2, \dots, 6$?

- 3 Show that the sum of the squares of the sides of a quadrilateral is at least the sum of the squares of the diagonals. When does equality hold?

- 4 For $n \neq 0$, let $f(n)$ be the largest k such that 3^k divides n . If M is a set of $n > 1$ integers, show that the number of possible values for $f(m - n)$, where m, n belong to M cannot exceed $n - 1$.

- 5 Let a, b be non-zero integers. Let $m(a, b)$ be the smallest value of $\cos ax + \cos bx$ (for real x). Show that for some r , $m(a, b) \leq r < 0$ for all a, b .
