www.artofproblemsolving.com/community/c1971601
by parmenides51

1 Find all integers m, n such that $m^3 = n^3 + n$.

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2 Show that $\tan \frac{\pi}{3n}$ is irrational for all positive integers n.

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3 $a_1 \leq a_2 \leq ... \leq a_n$ is a sequence of reals $b_1, _b2, b_3, ..., b_n$ is any rearrangement of the sequence $B_1 \leq B_2 \leq ... \leq B_n$. Show that $\sum a_i b_i \leq \sum a_i B_i$.

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4 Define $g(x)$ as the largest value of$|y^2 - xy|$ for y in $[0, 1]$. Find the minimum value of g (for real x).

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5 Let $N = a_1 a_2 ... a_n$ in binary. Show that if $a_1 - a_2 + a_3 - ... + (-1)^{n-1} a_n = 0$ mod 3, then $N = 0$ mod 3.

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6 Given $3n$ points in the plane, no three collinear, is it always possible to form n triangles (with vertices at the points), so that no point in the plane lies in more than one triangle?
