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by parmenides51

1 Show that infinitely many positive integers cannot be written as a sum of three fourth powers of integers.

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2 6 open disks in the plane are such that the center of no disk lies inside another. Show that no point lies inside all 6 disks.

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3 A polynomial with integer coefficients takes the value 5 at five distinct integers. Show that it does not take the value 9 at any integer.

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4 Let $p(x) = (x - x_1)(x - x_2)(x - x_3)$, where x_1, x_2 and x_3 are real. Show that $p(x)p''(x) \le p'(x)^2$ for all x.

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5 A 3×1 paper rectangle is folded twice to give a square side 1. The square is folded along a diagonal to give a right-angled triangle. A needle is driven through an interior point of the triangle, making 6 holes in the paper. The paper is then unfolded. Where should the point be in order to maximise the smallest distance between any two holes?

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6 Show that $\frac{(n-m)!}{m!} \le \left(\frac{n}{2} + \frac{1}{2}\right)^{n-2m}$ for positive integers m, n with $2m \le n$.
