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by parmenides51

1 Show that

$$\left(1+a+a^2\right)^2 < 3\left(1+a^2+a^4\right)$$

for real $a \neq 1$.

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2 An arbitrary number of lines divide the plane into regions. Show that the regions can be colored red and blue so that neighboring regions have different colors.

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3 A table is covered by 15 pieces of paper. Show that we can remove 7 pieces so that the remaining 8 cover at least $8/15$ of the table.

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4 Find

$$\frac{65533^3+65534^3+65535^3+65536^3+65537^3+65538^3+65539^3}{32765\cdot 32766+32767\cdot 32768+32768\cdot 32769+32770\cdot 32771}$$

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5 Show that

$$\max_{|x|\leq t}|1-a\cos x| \geq \tan^2\frac{t}{2}$$

for a positive and $t \in (0, \frac{\pi}{2})$.

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6 99 cards each have a label chosen from $1, 2, \ldots, 99$, such that no (non-empty) subset of the cards has labels with total divisible by 100. Show that the labels must all be equal.
