www.artofproblemsolving.com/community/c1974564
by parmenides51

1 Find the largest real number a such that

$$\begin{cases} x - 4y = 1 \\ ax + 3y = 1 \end{cases}$$

has an integer solution.

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2 A rectangular grid of streets has m north-south streets and n east-west streets. For which $m, n > 1$ is it possible to start at an intersection and drive through each of the other intersections just once before returning to the start?

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3 A steak temperature 5° is put into an oven. After 15 minutes, it has temperature 45°. After another 15 minutes it has temperature 77°. The oven is at a constant temperature. The steak changes temperature at a rate proportional to the difference between its temperature and that of the oven. Find the oven temperature.

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4 Put $x = \log_{10} 2$, $y = \log_{10} 3$. Then $15 < 16$ implies $1 - x + y < 4x$, so $1 + y < 5x$.
Derive similar inequalities from $80 < 81$ and $243 < 250$. Hence show that

$$0.47 < \log_{10} 3 < 0.482.$$

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5 Show that

$$\int\limits_0^1 \frac{1}{(1+x)^n} dx > 1 - \frac{1}{n}$$

for all positive integers n.

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6 $a_1, a_2, a_3, \dots$ and $b_1, b_2, b_3, \dots$ are sequences of positive integers. Show that we can find $m < n$ such that $a_m \le a_n$ and $b_m \le b_n$.
