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by parmenides51

1 A is the point $(1,0)$, L is the line $y = kx$ (where $k > 0$). For which points $P(t, 0)$ can we find a point Q on L such that AQ and QP are perpendicular?

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2 Is there a positive integer n such that the fractional part of

$$\left(3+\sqrt{5}\right)^n > 0.99?$$

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3 Show that

$$a^n + b^n + c^n \geq ab^{n-1} + bc^{n-1} + ca^{n-1}$$

for real $a, b, c \geq 0$ and n a positive integer.

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4 P_1, P_2, P_3, Q_1, Q_2, Q_3 are distinct points in the plane. The distances P_1Q_1, P_2Q_2, P_3Q_3 are equal. P_1P_2 and Q_2Q_1 are parallel (not antiparallel), similarly P_1P_3 and Q_3Q_1, and P_2P_3 and Q_3Q_2. Show that P_1Q_1, P_2Q_2 and P_3Q_3 intersect in a point.

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5 Show that n divides $2^n + 1$ for infinitely many positive integers n.

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6 $f(x)$ is defined for $0 \leq x \leq 1$ and has a continuous derivative satisfying $|f'(x)| \leq C|f(x)|$ for some positive constant C. Show that if $f(0) = 0$, then $f(x) = 0$ for the entire interval.
