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by parmenides51

1 In a tournament every team plays every other team just once. Each game is won by one of the teams (there are no draws). Each team loses at least once. Show that there must be three teams A, B, C such that A beat B, B beat C and C beat A.

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2 For which real a are there distinct reals x, y such that $x = a - y^2$ and $y = a - x^2$?

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3 If a, b, c are rational, show that

$$\frac{1}{(b-c)^2} + \frac{1}{(c-a)^2} + \frac{1}{(a-b)^2}$$

is the square of a rational.

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4 A number is placed in each cell of an $n \times n$ board so that the following holds:
(A) the cells on the boundary all contain 0;
(B) other cells on the main diagonal are each1 greater than the mean of the numbers to the left and right;
(C) other cells are the mean of the numbers to the left and right.
Show that (B) and (C) remain true if "left and right" is replaced by "above and below".

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5 $f(x)$ is defined for $x \geq 0$ and has a continuous derivative. It satisfies $f(0) = 1$, $f'(0) = 0$ and $(1 + f(x))f''(x) = 1 + x$. Show that f is increasing and that $f(1) \leq 4/3$.

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6 Show that there are only finitely many integral solutions to

$$3^m - 1 = 2^n$$

and find them.
