www.artofproblemsolving.com/community/c1974573
by parmenides51

1 p is a prime. Find the largest integer d such that p^d divides $p^4!$.

2 There is a point inside an equilateral triangle side d whose distance from the vertices is $3, 4, 5$. Find d.

3 Show that the only integral solution to

$$\begin{cases} xy + yz + zx = 3n^2 - 1 \\ x + y + z = 3n \end{cases}$$

with $x \geq y \geq z$ is $x = n + 1$, $y = n$, $z = n - 1$.

4 Show that if

$$\frac{\cos x}{\cos y} + \frac{\sin x}{\sin y} = -1$$

then

$$\frac{\cos^3 y}{\cos x} + \frac{\sin^3 y}{\sin x} = 1$$

5 The numbers $1, 2, 3, ..., 64$ are written in the cells of an 8×8 board (in some order, one per cell). Show that at least four 2×2 squares have sum greater than 100.

6 Show that there are positive reals a, b, c such that

$$\begin{cases} a^2 + b^2 + c^2 > 2 \\ a^3 + b^3 + c^3 < 2 \\ a^4 + b^4 + c^4 > 2 \end{cases}$$