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by parmenides51

1 Show that $\log_{10} 2$ is irrational.

2 $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and $b_1, b_2, b_3, b_4, b_5, b_6, b_7$ are two permutations of $1, 2, 3, 4, 5, 6, 7$. Show that $|a_1 - b_1|, |a_2 - b_2|, |a_3 - b_3|, |a_4 - b_4|, |a_5 - b_5|, |a_6 - b_6|, |a_7 - b_7|$ are not all different.

3 Let $T(n)$ be the number of dissimilar (non-degenerate) triangles with all side lengths integral and $\le n$. Find $T(n+1) - T(n)$.

4 The functions f and g are positive and continuous. f is increasing and g is decreasing. Show that
$$\int\limits_0^1 f(x)g(x)dx \le \int\limits_0^1 f(x)g(1-x)dx$$

5 A *word* is a string of the symbols a, b which can be formed by repeated application of the following:
(1) ab is a word;
(2) if X and Y are words, then so is XY;
(3) if X is a word, then so is aXb.
How many words have 12 letters?

6 Find the smallest constant c such that for every 4 points in a unit square there are two a distance $\le c$ apart.