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by parmenides51

1 Let $N = 11\cdots 122\cdots 25$, where there are n 1s and $n+1$ 2s. Show that N is a perfect square.

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2 Does

$$\begin{cases} x^y = z \\ y^z = x \\ z^x = y \end{cases}$$

have any solutions in positive reals apart from $x = y = z = 1$?

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3 Find all polynomials $p(x)$ of degree 5 such that $p(x)+1$ is divisible by $(x-1)^3$ and $p(x)-1$ is divisible by $(x+1)^3$.

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4 A cube side 5 is divided into 125 unit cubes. N of the small cubes are black and the rest white. Find the smallest N such that there must be a row of 5 black cubes parallel to one of the edges of the large cube.

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5 ABC is a triangle. X, Y, Z lie on BC, CA, AB respectively. Show that area XYZ cannot be smaller than each of area AYZ, area BZX, area CXY.

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6 Show that there are infinitely many triangles with side lengths a, b, c, where a is a prime, b is a power of 2 and c is the square of an odd integer.
