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- 1 How many solutions does

$$x^2 - [x^2] = (x - [x])^2$$

have satisfying $1 \leq x \leq n$?

- 2 Show that

$$abc \geq (a + b - c)(b + c - a)(c + a - b)$$

for positive reals a, b, c .

- 3 Show that there is a point P inside the quadrilateral $ABCD$ such that the triangles PAB, PBC, PCD, PDA have equal area. Show that P must lie on one of the diagonals.
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- 4 ABC is a triangle with $AB = 33, AC = 21$ and $BC = m$, an integer. There are points D, E on the sides AB, AC respectively such that $AD = DE = EC = n$, an integer. Find m .
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- 5 Each point in a 12×12 array is colored red, white or blue. Show that it is always possible to find 4 points of the same color forming a rectangle with sides parallel to the sides of the array.
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- 6 Show that

$$(2a - 1) \sin x + (1 - a) \sin(1 - a)x \geq 0$$

for $0 \leq a \leq 1$ and $0 \leq x \leq \pi$.
