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by parmenides51

1 The positive integers are grouped as follows: $1, 2+3, 4+5+6, 7+8+9+10, \ldots$. Find the value of the n-th sum.

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2 Show that

$$\cos x^2 + \cos y^2 - \cos xy < 3$$

for reals x, y.

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3 The systems of equations

$$\begin{cases} 2x_1 - x_2 = 1 \\ -x_1 + 2x_2 - x_3 = 1 \\ -x_2 + 2x_3 - x_4 = 1 \\ -x_3 + 3x_4 - x_5 = 1 \\ \cdots\cdots\cdots\cdots \\ -x_{n-2} + 2x_{n-1} - x_n = 1 \\ -x_{n-1} + 2x_n = 1 \end{cases}$$

has a solution in positive integers x_i. Show that n must be even.

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4 C, C' are concentric circles with radii R, R'. A rectangle has two adjacent vertices on C and the other two vertices on C'. Find its sides if its area is as large as possible.

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5 Show that a unit square can be covered with three equal disks with radius less than $\frac{1}{\sqrt{2}}$. What is the smallest possible radius?

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6 Show that the only real solution to

$$\begin{cases} x(x+y)^2 = 9 \\ x(y^3 - x^3) = 7 \end{cases}$$

is $x = 1$, $y = 2$.
