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by parmenides51

1 Let A and B be two points inside a circle C. Show that there exists a circle that contains A and B and lies completely inside C.

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2 The squares in a 3×7 grid are colored either blue or yellow. Consider all $m \times n$ rectangles in this grid, where $m \in \{2, 3\}$, $n \in \{2, ..., 7\}$. Prove that at least one of these rectangles has all four corner squares the same color.

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3 Prove that if a, b are positive numbers, then

$$\left(\frac{a+1}{b+1}\right)^{b+1} \geq \left(\frac{a}{b}\right)^{b}$$

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4 Find all positive integers p and q such that all the roots of the polynomial $(x^2-px+q)(x^2-qx+p)$ are positive integers.

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5 Solve in natural numbers a, b, c the system

$$\begin{cases} a^3 - b^3 - c^3 = 3abc \\ a^2 = 2(a+b+c) \end{cases}$$

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6 Assume $a_1, a_2, ..., a_{14}$ are positive integers such that $\sum_{i=1}^{14} 3^{a_i} = 6558$.
Prove that the numbers $a_1, a_2, ..., a_{14}$ consist of the numbers $1, ..., 7$, each taken twice.
