

AoPS Community

1986 Swedish Mathematical Competition

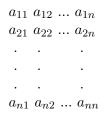
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by parmenides51

- 1 Show that the polynomial $x^6 x^5 + x^4 x^3 + x^2 x + \frac{3}{4}$ has no real zeroes.
- **2** The diagonals AC and BD of a quadrilateral ABCD intersect at O. If S_1 and S_2 are the areas of triangles AOB and COD and S that of ABCD, show that $\sqrt{S_1} + \sqrt{S_2} \le \sqrt{S}$. Prove that equality holds if and only if AB and CD are parallel.
- **3** Let $N \ge 3$ be a positive integer. For every pair (a, b) of integers with $1 \le a < b \le N$ consider the quotient q = b/a. Show that the pairs with q < 2 are equally numbered as those with q > 2.
- **4** Prove that x = y = z = 1 is the only positive solution of the system

$$\left\{ \begin{array}{l} x+y^2+z^3=3\\ y+z^2+x^3=3\\ z+x^2+y^3=3 \end{array} \right.$$

5 In the arrangement of pn real numbers below, the difference between the greatest and smallest numbers in each row is at most d, d > 0.



Prove that, when the numbers in each column are rearranged in decreasing order, the difference between the greatest and smallest numbers in each row will still be at most d.

6 The interval [0, 1] is covered by a finite number of intervals. Show that one can choose a number of these intervals which are pairwise disjoint and have the total length at least 1/2.

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