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by parmenides51

1   How many positive integers have square less than 10^7?

2   The squares of a chessboard have side 4. What is the circumference of the largest circle that can be drawn entirely on the black squares of the board?

3   What is the remainder on dividing $1234^{567} + 89^{1011}$ by 12?

4   Given the real number k, find all differentiable real-valued functions $f(x)$ defined on the reals such that $f(x + y) = f(x) + f(y) + f(kxy)$ for all x, y.

5   A road has constant width. It is made up of finitely many straight segments joined by corners, where the inner corner is a point and the outer side is a circular arc. The direction of the straight sections is always between NE (45°) and SSE ($1571/2^o$). A person wishes to walk along the side of the road from point A to point B on the same side. He may only cross the street perpendicularly. What is the shortest route?

[figure missing]

6   The real-valued function $f(x)$ is defined on the reals. It satisfies $|f(x)| \leq A$, $|f''(x)| \leq B$ for some positive A, B (and all x). Show that $|f'(x)| \leq C$, for some fixed C, which depends only on A and B. What is the smallest possible value of C?