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by parmenides51

**1** An integer  $x$  has the property that the sums of the digits of  $x$  and of  $3x$  are the same. Prove that  $x$  is divisible by 9.

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**2** A railway line is divided into ten sections by the stations  $A, B, C, D, E, F, G, H, I, J, K$ . The length of each section is an integer number of kilometers and the distance between  $A$  and  $K$  is 56 km. A trip along two successive sections never exceeds 12 km, but a trip along three successive sections is at least 17 km. What is the distance between  $B$  and  $G$ ?

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**3** Assume that  $a$  and  $b$  are integers. Prove that the equation  $a^2 + b^2 + x^2 = y^2$  has an integer solution  $x, y$  if and only if the product  $ab$  is even.

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**4** To each pair of nonzero real numbers  $a$  and  $b$  a real number  $a * b$  is assigned so that  $a * (b * c) = (a * b)c$  and  $a * a = 1$  for all  $a, b, c$ . Solve the equation  $x * 36 = 216$ .

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**5** A triangle with sides  $a, b, c$  and perimeter  $2p$  is given. Is possible, a new triangle with sides  $p - a, p - b, p - c$  is formed. The process is then repeated with the new triangle. For which original triangles can this process be repeated indefinitely?

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**6** For real numbers  $a$  and  $b$  define  $f(x) = \frac{1}{ax+b}$ . For which  $a$  and  $b$  are there three distinct real numbers  $x_1, x_2, x_3$  such that  $f(x_1) = x_2, f(x_2) = x_3$  and  $f(x_3) = x_1$ ?

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