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by parmenides51

1 $x\sqrt{8}+\frac{1}{x\sqrt{8}}=\sqrt{8}$ has two real solutions x_1, x_2. The decimal expansion of x_1 has the digit 6 in place 1994. What digit does x_2 have in place 1994?

2 In the triangle ABC, the medians from B and C are perpendicular. Show that $\cot B+\cot C \geq \frac{2}{3}$.

3 The vertex B of the triangle ABC lies in the plane P. The plane of the triangle meets the plane in a line L. The angle between L and AB is a, and the angle between L and BC is b. The angle between the two planes is c. Angle ABC is 90^o. Show that $\sin^2 c = \sin^2 a + \sin^2 b$.
https://cdn.artofproblemsolving.com/attachments/9/e/c0608e5408fd27a5f907a3488cce7dc2af695.png

4 Find all integers m, n such that $2n^3 - m^3 = mn^2 + 11$.

5 The polynomial $x^k + a_1x^{k-1} + a_2x^{k-2} + ... + a_k$ has k distinct real roots. Show that $a_1^2 > \frac{2ka_2}{k-1}$.

6 Let N be the set of non-negative integers. The function $f : N \to N$ satisfies $f(a + b) = f(f(a)+b)$ for all a, b and $f(a+b) = f(a)+f(b)$ for $a+b < 10$. Also $f(10) = 1$. How many three digit numbers n satisfy $f(n) = f(N)$, where N is the "tower" $2, 3, 4, 5$, in other words, it is 2^a, where $a = 3^b$, where $b = 4^5$?