

www.artofproblemsolving.com/community/c1978758

by parmenides51

- 1 Find all positive integers a, b, c , such that $(8a - 5b)^2 + (3b - 2c)^2 + (3c - 7a)^2 = 2$.

- 2 ABC is a triangle. Show that $c \geq (a + b) \sin \frac{C}{2}$.

- 3 A cube side 5 is made up of unit cubes. Two small cubes are *adjacent* if they have a common face. Can we start at a cube adjacent to a corner cube and move through all the cubes just once? (The path must always move from a cube to an adjacent cube).

- 4 $ABCD$ is a quadrilateral with $\angle A = 90^\circ$, $AD = a$, $BC = b$, $AB = h$, and area $\frac{(a+b)h}{2}$. What can we say about $\angle B$?

- 5 Show that for any $n > 5$ we can find positive integers x_1, x_2, \dots, x_n such that $\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} = \frac{1997}{1998}$. Show that in any such equation there must be two of the n numbers with a common divisor (> 1).

- 6 Show that for some $c > 0$, we have $|\sqrt[3]{2} - \frac{m}{n}| > \frac{c}{n^3}$ for all integers m, n with $n \geq 1$.