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by parmenides51

- 1 Solve  $|||||x^2 - x - 1| - 2| - 3| - 4| - 5| = x^2 + x - 30$ .

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- 2 Circle  $C$  center  $O$  touches externally circle  $C'$  center  $O'$ . A line touches  $C$  at  $A$  and  $C'$  at  $B$ .  $P$  is the midpoint of  $AB$ . Show that  $\angle OPO' = 90^\circ$ .

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- 3 Find non-negative integers  $a, b, c, d$  such that  $5^a + 6^b + 7^c + 11^d = 1999$ .

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- 4 An equilateral triangle of side  $x$  has its vertices on the sides of a square side 1. What are the possible values of  $x$ ?

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- 5  $x_i$  are non-negative reals.  $x_1 + x_2 + \dots + x_n = s$ . Show that  $x_1x_2 + x_2x_3 + \dots + x_{n-1}x_n \leq \frac{s^2}{4}$ .

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- 6  $S$  is any sequence of at least 3 positive integers. A move is to take any  $a, b$  in the sequence such that neither divides the other and replace them by  $\gcd(a, b)$  and  $\text{lcm}(a, b)$ . Show that only finitely many moves are possible and that the final result is independent of the moves made, except possibly for order.

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