www.artofproblemsolving.com/community/c1978784
by parmenides51

1 Let $d_1, d_2, ..., d_k$ be the positive divisors of $n = 1990!$. Show that $\sum \frac{d_i}{\sqrt{n}} = \sum \frac{\sqrt{n}}{d_i}$.

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2 The points $A_1, A_2, .., A_{2n}$ are equally spaced in that order along a straight line with $A_1A_2 = k$. P is chosen to minimise $\sum PA_i$. Find the minimum.

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3 Find all a, b such that $\sin x + \sin a \geq b \cos x$ for all x.

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4 $ABCD$ is a quadrilateral. The bisectors of $\angle A$ and $\angle B$ meet at E. The line through E parallel to CD meets AD at L and BC at M. Show that $LM = AL + BM$.

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5 Find all monotonic positive functions $f(x)$ defined on the positive reals such that $f(xy)f\left(\frac{f(y)}{x}\right) = 1$ for all x, y.

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6 Find all positive integers m, n such that $\frac{117}{158} > \frac{m}{n} > \frac{97}{131}$ and $n \leq 500$.
