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by parmenides51

1 Find all positive integers m, n such that $\frac{1}{m} + \frac{1}{n} - \frac{1}{mn} = \frac{2}{5}$.

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2 x, y are positive reals such that $x - \sqrt{x} \le y - 1/4 \le x + \sqrt{x}$. Show that $y - \sqrt{y} \le x - 1/4 \le y + \sqrt{y}$.

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3 The sequence $x_0, x_1, x_2, ...$ is defined by $x_0 = 0$, $x_{k+1} = [(n - \sum_0^k x_i)/2]$. Show that $x_k = 0$ for all sufficiently large k and that the sum of the non-zero terms x_k is $n - 1$.

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4 $x_1, x_2, ..., x_8$ is a permutation of $1, 2, ..., 8$. A move is to take x_3 or x_8 and place it at the start to from a new sequence. Show that by a sequence of moves we can always arrive at $1, 2, ..., 8$.

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5 Show that there are infinitely many odd positive integers n such that in binary n has more 1s than n^2.

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6 Given any triangle, show that we can always pick a point on each side so that the three points form an equilateral triangle with area at most one quarter of the original triangle.
