

www.artofproblemsolving.com/community/c1978804

by parmenides51

- 1 Is $\frac{19^{92}-91^{29}}{90}$ an integer?

- 2 The squares in a 9×9 grid are numbered from 11 to 99, where the first digit is the row and the second the column. Each square is colored black or white. Squares 44 and 49 are black. Every black square shares an edge with at most one other black square, and each white square shares an edge with at most one other white square. What color is square 99?

- 3 Solve: $2x_1 - 5x_2 + 3x_3 \geq 0$ $2x_2 - 5x_3 + 3x_4 \geq 0$... $2x_{23} - 5x_{24} + 3x_{25} \geq 0$ $2x_{24} - 5x_{25} + 3x_1 \geq 0$ $2x_{25} - 5x_1 + 3x_2 \geq 0$

- 4 Find all positive integers a, b, c such that $a < b$, $a < 4c$, and $bc^3 \leq ac^3 + b$.

- 5 A triangle has sides a, b, c with longest side c , and circumradius R . Show that if $a^2 + b^2 = 2cR$, then the triangle is right-angled.

- 6 $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on a straight line and on the curve $y^2 = x^3$. Show that $\frac{x_1}{y_1} + \frac{x_2}{y_2} + \frac{x_3}{y_3} = 0$.