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– Category D

Problem 1 Mark six points in a plane so that any three of them are vertices of a nondegenerate isosceles triangle.

Problem 2 Find all positive integers n so that both n and $n+100$ have odd numbers of divisors.

Problem 3 Some students of a group were friends of some others. One day all students of the group take part in a picnic. During the picnic some friends had a quarrel with each other, but some other students became friends. After the picnic, the number of friends for each student changed by 1. Prove that the number of students in the group was even.

Problem 4 Given a triangle ABC, let K be the midpoint of AB and L be the point on the side AC such that $AL = LC + CB$. Show that if $\angle KLB = 90^o$ then $AC = 3CB$ and conversely, if $AC = 3CB$ then $\angle KLB = 90^o$.

Problem 5 Two circles touch in M, and lie inside a rectangle $ABCD$. One of them touches the sides AB and AD, and the other one touches AD, BC, CD. The radius of the second circle is four times that of the first circle. Find the ratio in which the common tangent of the circles in M divides AB and CD.

Problem 6 Let p and q be distinct positive integers. Prove that at least one of the equations $x^2+px+q=0$ and $x^2+qx+p=0$ has a real root.

Problem 7 The expression $1\oplus 2\oplus 3\oplus 4\oplus 5\oplus 6\oplus 7\oplus 8\oplus 9$ is written on a blackboard. Bill and Peter play the following game. They replace $\oplus$ by $+$ or $\cdot$, making their moves in turn, and one of them can use only $+$, while the other one can use only $\cdot$. At the beginning, Bill selects the sign he will use, and he tries to make the result an even number. Peter tries to make the result an odd number. Prove that Peter can always win.

The expression $1*2*3*4*5*6*7*8*9$ is written on a blackboard. Bill and Peter play the following game. They replace $*$ by $+$ or $\cdot$, making their moves in turn, and one of them can use only $+$, while the other one can use only $\cdot$. At the beginning Bill selects the sign he will use, and he tries to make the result an even number. Peter tries to make the result an odd number. Prove that Peter can always win.

Problem 8 Five numbers 1,2,3,4,5 are written on a blackboard. A student may erase any two of the numbers a and b on the board and write the numbers a+b and ab replacing them. If this operation is performed repeatedly, can the numbers 21,27,64,180,540 ever appear on the board?