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– 1st Grade

Problem 1 None of the positive integers k, m, n are divisible by 5. Prove that at least one of the numbers $k^2 - m^2, m^2 - n^2, n^2 - k^2$ is divisible by 5.

Problem 2 Tina wrote a positive number on each of five pieces of paper. She did not say which numbers she wrote, but revealed their pairwise sums instead: $17, 20, 28, 14, 42, 36, 28, 39, 25, 31$. Which numbers did she write?

Problem 3 For an arbitrary point P on a given segment AB, two isosceles right triangles APQ and PBR with the right angles at Q and R are constructed on the same side of the line AB. Prove that the distance from the midpoint M of QR to the line AB does not depend on the choice of P.

Problem 4 Andrej and Barbara play the following game with two strips of newspaper of length a and b. They alternately cut from any end of any of the strips a piece of length d. The player who cannot cut such a piece loses the game. Andrej allows Barbara to start the game. Find out how the lengths of the strips determine the winner.

– 2nd Grade

Problem 1 Determine all positive integers a, b, c such that $ab + ac + bc$ is a prime number and

$$\frac{a+b}{a+c} = \frac{b+c}{b+a}.$$

Problem 2 Let $p(n)$ denote the product of decimal digits of a positive integer n. Computer the sum $p(1) + p(2) + \ldots + p(2001)$.

Problem 3 Let E and F be points on the side AB of a rectangle $ABCD$ such that $AE = EF$. The line through E perpendicular to AB intersects the diagonal AC at G, and the segments FD and BG intersect at H. Prove that the areas of the triangles FBH and GHD are equal.

Problem 4 Find the smallest number of squares on an 8×8 board that should be colored so that every L-tromino on the board contains at least one colored square.

– 3rd Grade

Problem 1 (a) Prove that $\sqrt{n+1}-\sqrt{n}<\frac{1}{2\sqrt{n}}<\sqrt{n}-\sqrt{n-1}$ for all $n\in\mathbb{N}$.
(b) Prove that the integer part of the sum $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{m^2}}$, where $m\in\mathbb{N}$, is either $2m-2$ or $2m-1$.

Problem 2 Find all rational numbers r such that the equation $rx^2+(r+1)x+r=1$ has integer solutions.

Problem 3 A point D is taken on the side BC of an acute-angled triangle ABC such that $AB=AD$. Point E on the altitude from C of the triangle is such that the circle k_1 with center E is tangent to the line AD at D. Let k_2 be the circle through C that is tangent to AB at B. Prove that A lies on the line determined by the common chord of k_1 and k_2.

Problem 4 Cross-shaped tiles are to be placed on a 8×8 square grid without overlapping. Find the largest possible number of tiles that can be placed.
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– 4th Grade

Problem 1 Let a,b,c,d,e,f be positive numbers such that a,b,c,d is an arithmetic progression, and a,e,f,d is a geometric progression. Prove that $bc\geq ef$.

Problem 2 Find all prime numbers p for which $3^p-(p+2)^2$ is also prime.

Problem 3 Let D be the foot of the altitude from A in a triangle ABC. The angle bisector at C intersects AB at a point E. Given that $\angle CEA=\frac{\pi}{4}$, compute $\angle EDB$.

Problem 4 Let $n\geq 4$ points on a circle be denoted by 1 through n. A pair of two nonadjacent points denoted by a and b is called regular if all numbers on one of the arcs determined by a and b are less than a and b. Prove that there are exactly $n-3$ regular pairs.
