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– 1st Grade

Problem 1 Two three-digit numbers are given. The hundreds digit of each of them is equal to the units digit of the other. Find these numbers if their difference is 297 and the sum of digits of the smaller number is 23.

Problem 2 Find all integers x,y such that $2x+3y=185$ and $xy>x+y$.

Problem 3 The incircle of a right triangle ABC touches the hypotenuse AB at a point D. Show that the area of $\triangle ABC$ equals $AD\cdot DB$.

Problem 4 On a mountain, three shepherds cyclically alternate shearing the same herd of sheep. The shepherds agreed to obey the following rules:
(i) Every day a sheep can be shorn* on one side only;
(ii) Every day at least one sheep must be shorn;
(iii) No two days the same group of sheep can be shorn.
The shepherd who first breaks the agreement will have to accompany the herd in the valley next fall. Can anyone of the shepherds shear the sheep in such a way to make sure that he will avoid this punishment?

*shorn is the past tense of shear

– 2nd Grade

Problem 1 Prove that the product of three consecutive positive integers is never a perfect square.

Problem 2 Three unit vectors a,b,c are given on the plane. Prove that one can choose the signs in the expression $x=\pm a\pm b\pm c$ so as to obtain a vector x with $|x|\le\sqrt{2}$.

Problem 3 A semicircle with diameter AB is given. Two non-intersecting circles k_1 and k_2 with different radii touch the diameter AB and touch the semicircle internally at C and D, respectively. An interior common tangent t of k_1 and k_2 touches k_1 at E and k_2 at F. Prove that the lines CE and DF intersect on the semicircle.

Problem 4 Three integers are written on a blackboard. At every step one of them is erased and the sum of the other two decreased by 1 is written instead. Is it possible to obtain the numbers $17,75,91$ if the three initial numbers were: **(a)** $2,2,2$; **(b)** $3,3,3$?

– 3rd Grade

Problem 1 What is the smallest possible value of $|12^m - 5^n|$, where m and n are positive integers?

Problem 2 Consider the polynomial $p(x) = x^{1999} + 2x^{1998} + 3x^{1997} + \ldots + 2000$. Find a nonzero polynomial whose roots are the reciprocal values of the roots of $p(x)$.

Problem 3 Let O be the circumcenter of a triangle ABC, P be the midpoint of AO, and Q be the midpoint of BC. If $\angle ABC = 4\angle OPQ$ and $\angle ACB = 6\angle OPQ$, compute $\angle OPQ$.

Problem 4 A pawn is put on each of $2n$ arbitrary selected cells of an $n \times n$ board ($n > 1$). Prove that there are four cells that are marked with pawns and whose centers form a parallelogram.

– 4th Grade

Problem 1 Let $r_1, r_2, \ldots, r_m$ be positive rational numbers with a sum of 1. Find the maximum values of the function $f : \mathbb{N} \to \mathbb{Z}$ defined by

$$f(n) = n - \lfloor r_1 n \rfloor - \lfloor r_2 n \rfloor - \ldots - \lfloor r_m n \rfloor$$

Problem 2 The numbers $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{1999}$ are written on a blackboard. In every step, we choose two of them, say a and b, erase them, and write the number $ab + a + b$ instead. This step is repeated until only one number remains. Can the last remaining number be equal to 2000?

Problem 3 A section of a rectangular parallelepiped by a plane is a regular hexagon. Prove that this parallelepiped is a cube.

Problem 4 Let be given three-element subsets $A_1, A_2, \ldots, A_6$ of a six-element set X. Prove that the elements of X can be colored with two colors in such a way that none of the given subsets are monochromatic.