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by parmenides51

1 Solve the following system

$$\begin{cases} xyzu - x^3 = 9 \\ x + yz = \dfrac{3}{2}u \end{cases}$$

in positive integers x, y, z and u.

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2 A number of flowers are distributed between n persons so that the first of them, Andreas, gets one flower, the other gets two flowers, the third gets three flowers, etc., to n-th person who gets n flowers. Andreas then walks around shaking hands with each other of the others, in any order. In order to do so, he receives a flower from everyone which he hangs on to and which has more flowers than himself at the moment they shake hands. Which is the smallest number of flowers Andreas can have after shaking hands with everyone?

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3 Let α, β, γ be the angles of a triangle. If a, b, c are the side length of the triangle and R is the circumradius, show that

$$\cot\alpha + \cot\beta + \cot\gamma = \frac{R\left(a^2+b^2+c^2\right)}{abc}$$

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4 There are a number of arcs on the edge of a circular disk. Each pair of arcs has the least one point in common. Show that on the circle you can choose two diametrical opposites points such that each arc contains at least one of these two points.

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5 Anna and Brian play a game where they put the domino tiles (of size 2×1) in a boards composed of $n\times 1$ boxes. Tiles must be placed so that they cover exactly two boxes. Players take turnslaying each tile and the one laying last tile wins. They play once for each n, where $n = 2, 3, \ldots, 2007$. Show that Anna wins at least 1505 of the games if she always starts first and they both always play optimally, ie if they do their best to win in every move.

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6 In the plane, a triangle is given. Determine all points P in the plane such that each line through P that divides the triangle into two parts with the same area must pass through one of the vertices of the triangle.
