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by parmenides51

1 A rhombus is inscribed in a convex quadrilateral. The sides of the rhombus are parallel with the diagonals of the quadrilateral, which have the lengths d_1 and d_2. Calculate the length of side of the rhombus , expressed in terms of d_1 and d_2.

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2 Determine the smallest integer $n \geq 3$ with the property that you can choose two of the numbers $1, 2, \ldots, n$ in such a way that their product is equal to the sum of the other $n-2$ languages. What are the two numbers?

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3 The function $f(x)$ has the property that $\frac{f(x)}{x}$ is increasing for $x > 0$. Show that

$$f(x) + f(y) \leq f(x+y) \quad , \quad \text{for all } x, y > 0$$

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4 A convex n-side polygon has angles $v_1, v_2, \ldots, v_n$ (in degrees), where all v_k ($k = 1, 2, \ldots, n$) are positive integers divisible by 36.
(a) Determine the largest n for which this is possible.
(b) Show that if $n > 5$, two of the sides of the n-polygon must be parallel.

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5 Anna and Orjan play the following game: they start with a positive integer $n > 1$, Anna writes it as the sum of two other positive integers, $n = n_1 + n_2$. Orjan deletes one of them, n_1 or n_2. If the remaining number is larger than 1, the process is repeated, i.e. Anna writes it as the sum of two positive integers, $n_3 + n_4$, Orjan deletes one of them etc. The game ends when the last number is 1. Orjan is the winner if there are two equal numbers among the numbers he has deleted, otherwise Anna wins. Who is winning the game if n = 2008 and they both play optimally?

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6 A *sum decomposition* of the number 100 is given by a positive integer n and n positive integers $x_1 < x_2 < \cdots < x_n$ such that $x_1 + x_2 + \cdots + x_n = 100$. Determine the largest possible value of the product $x_1 x_2 \cdots x_n$, and n , as $x_1, x_2, \ldots, x_n$ vary among all sum decompositions of the number 100.
