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- 1 Five square carpets have been bought for a square hall with a side of 6 m, two with the side 2 m, one with the side 2.1 m and two with the side 2.5 m. Is it possible to place the five carpets so that they do not overlap in any way each other? The edges of the carpets do not have to be parallel to the cradles in the hall.

- 2 Find all real solutions of the equation

$$(1 + x^2)(1 + x^3)(1 + x^5) = 8x^5$$

- 3 An urn contain a number of yellow and green balls. You extract two balls from the urn (without adding them back) and calculate the probability of both balls being green. Can you choose the number of yellow and green balls such that this probability to be $\frac{1}{4}$?

- 4 Determine all integers solutions of the equation $x + x^3 = 5y^2$.

- 5 A semicircular arc and a diameter AB with a length of 2 are given. Let O be the midpoint of the diameter. On the radius perpendicular to the diameter, we select a point P at the distance d from the midpoint of the diameter O , $0 < d < 1$. A line through A and P intersects the semicircle at point C . Through point P we draw another line at right angle against AC that intersects the semicircle at point D . Through point C we draw a line l_1 , parallel to PD and then a line l_2 , through D parallel to PC . The lines l_1 and l_2 intersect at point E . Show that the distance between O and E is equal to $\sqrt{2 - d^2}$

- 6 On a table lie 289 coins that form a square array 17×17 . All coins are facing with the crown up. In one move, it is possible to reverse any five coins lying in a row: vertical, horizontal or diagonal. Is it possible that after a number of such moves, all the coins to be arranged with tails up?