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by parmenides51

- 1 Exists a triangle whose three altitudes have lengths 1, 2 and 3 respectively?

- 2 Consider the four lines $y = mx - k^2$ for different integer k . Let (x_i, y_i) , $i = 1, 2, 3, 4$ be four different points, such that each belongs to two different lines and on each line pass through just the two of them. Let $x_1 \leq x_2 \leq x_3 \leq x_4$. Show that $x_1 + x_4 = x_2 + x_3$ and $y_1 y_4 = y_2 y_3$.

- 3 Find all natural numbers $n \geq 1$ such that there is a polynomial $p(x)$ with integer coefficients for which $p(1) = p(2) = 0$ and where $p(n)$ is a prime number.

- 4 We create a sequence by setting $a_1 = 2010$ and requiring that $a_n - a_{n-1} \leq n$ and a_n is also divisible by n .
Show that $a_{100}, a_{101}, a_{102}, \dots$ form an arithmetic sequence.

- 5 Consider the number of triangles where the side lengths a, b, c satisfy $(a+b+c)(a+b-c) = 2b^2$. Determine the angles in the triangle for which the angle opposite to the side with the length a is as big as possible.

- 6 An infinite number of squares on an infinitely square grid paper are painted red. Show that you can draw a number of squares on the paper, with sides along the grid lines, such that:
 - (1) no square in the grid belongs to more than one square (an edge, on the other hand, may belong to more than one square)
 - (2) each red square is located in one of the squares and the number of red squares in such square is at least $1/5$ and at most $4/5$ of the number of squares in the square.

Ett ändligt antal rutor på ett oändligt rutat papper är målade röda. Visa att man på papperet kan rita in ett antal kvadrater, med sidor utefter rutnätets linjer, sådana att :

 - (1) ingen ruta i nätet tillhör mer än en kvadrat (en kant kan däremot tillhöra mer än en kvadrat),
 - (2) varje röd ruta ligger i någon av kvadraterna och antalet röda rutor i en sådan kvadrat är minst $1/5$ och högst $4/5$ av antalet rutor i kvadraten.

source (<http://www.mattetavling.se/wp-content/uploads/2011/01/Final10.pdf>)

PS. I always post the original wording when I doubt about my (using Google) translation.